# Optimal supercruise flight paths

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In this article we investigate the ship dynamics in supercruise within star systems. We identify the maximal velocity, which is the speed a ship attempts to reach when set to maximal throttle. We measure this velocity, and find that it depends on how far one is away from the nearest star. Thus, one can use geodesic equations to compute the optimal flight path. We simulate some trajectories, and finally give some guidelines as to how to move optimally in a star system.

## I. MOTIVATION

Modern civilisation rests on the frame shift drive (FSD). Without it, there would be no interstellar travel to speak of. Although there have been attempts at colonising the galaxy from Earth without faster-than-light travel[1], it can be safely asserted that our modern society relies on the FSD.

In its present day incarnation, the FSD is constructed solely by Sirius Inc, which uses its monopoly to keep the construction details of the drive as secret as possible. Therefore, despite its ubiquity, many of the features of the FSD are not entirely understood. Its use by the galactic community rests, ironically, on experience and hearsay, rather than precise technical specifications.

To overcome this problem, we attempt to learn more about the properties of the FSD by measurements and computational analysis. In this article, we discuss the optimal travel through a solar system by FSD. We are interested in the following question:

> **What is the supercruise flight path between two points in a star system that takes the least amount of time?**

This is, in fact, not a straight line, since a ship gets slower when closer to a star or planet. In the following article, we will investigate the details, and compute the optimal flight path, analytically and numerically.

In section II, we talk about ship dynamics and our general findings first. In section III we discuss the five different regions around a star, and how the maximal velocity of a ship behaves as function of distance to a star. This contains detailed measurements taken in a Cobra Mk III. In section IV we use this data to compute the quickest paths through a star system. While the optimal flight paths can be quite complicated, one can,

very roughly, sum them up in some broad rules for optimal flying:

> *Rule 1: Leave the region where $v = 0.33c$ as directly as possible!*
>
> *Rule 2: It is almost never recommendable to fly directly towards one's target in a straight line. Rather, give the nearest star a wide berth*

These rules are mere guidelines. For more specific rules, which apply for the different regions around a star, we refer to section IV.

We close the article with a summary of our findings.

## II. SHIP DYNAMICS IN SUPERCRUISE (SC)

A ship in supercruise (SC) can move much faster than light, based on original calculations from the late 20th century [1, 2]. It allows to reach even far away stations in mere minutes[2].

When setting the throttle to 100% in SC, the ship attempts to obtain the **maximal velocity** $v_{\max}$. Note that for ships with low acceleration, it will take some time to reach $v_{\max}$. When the ship in the meantime enters a region with larger $v_{\max}$, the ship will lag behind its maximal velocity for quite some time. Also note that $v_{\max}$ *is not actually the maximal speed a ship can have*: If the ship has low deceleration, it might not slow down quickly enough when $v_{\max}$ reduces. It will then be faster than its indicated $v_{\max}$ until it has slowed down sufficiently. The ship's computer usually indicates this by a "SLOW DOWN" message.

In our article, we are interested in $v_{\max}$, and how it changes. A first few – quite surprising – observations we made were:

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[1] See e.g. information about generation ships at `https://canonn.science/codex/cartographics-exploration/`.

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[2] The furthest station from the entry point of a system is Hutton Orbital Station in Alpha Centauri. Their remoteness makes for few visitors, which the owners of the station have tried to remedy by giving out a free Anaconda to any pilot who makes it to their station.

FIG. 1. Measurement of $v_{\max}$: Distance is $r = 77.3$, throttle at 10%, so $v_{\max} = 15.3$.

1. **$v_{\max}$ does not depend on the type of ship or the type of equipped frame FSD.** As far as we can tell, the maximal velocity of a ship is the same for the type Sidewinder, Cobra Mk III, Asp Explorer, and Lakon Type 9. This makes our discussion much more general than we had originally thought. Note that all of these ships have vastly different acceleration/deceleration, though. Our discussion is most precise for those ships with high acceleration/deceleration, since for those their actual speed will always be close to $v_{\max}$ when under full throttle.

2. **$v_{\max}$ does only depend on the distance r to the nearest stellar body.** We are only considering stellar bodies here, which means our results are applicable only for ships that do not come too close to planets. Planets, as well as the dynamics of planetary landing, will be treated in a future article. Also note that, technically, not the closest star counts, but the one which sets the lowest $v_{\max}$, as given by the formulas presented in the next sections. For most cases, this will, however, be the closest star. Details will appear in a future article.[3]

3. **$v_{\max}(r)$ does not depend on the type of star, or its mass.** In particular, the only important data is the *closest* star, not the most massive one.

Further stars have no influence on $v_{\max}(r)$. This rule comes with the caveat that it only holds for main sequence stars. [4]

### A. Measuring $v_{\max}$

For the rest of the article, we adopt the following convention:

> **Convention:** Throughout this article, speed $v$ is always measured in multiples of the speed of light $c \approx 3 \cdot 10^8 \mathrm{ms}^{-1}$. Furthermore, distance $r$ will generally be measured in light seconds $ls \approx 3 \cdot 10^8 \mathrm{m}$.

In order to measure $v_{\max}$ depending on $r$, we chose a system with only one star, which we took to be COL 285 SECTOR NG-F B11-2. The ship layout was a Cobra Mk III with A-grade FSD and A-grade thrusters. We set the throttle to 10%, in order to be as slow as possible, and

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[3] We thank CMDR bakwards for pointing this out to us.

[4] The dynamics for black holes seems to be slightly different, which might be due to gravitational time distortion. The curves for neutron stars / white dwarfs have not been tested thoroughly, due to the danger of flying in their immediate vicinity. Further scientific analysis is contingent on a funding grant for replacing destroyed ships and a lifetime's supply of remlok masks.
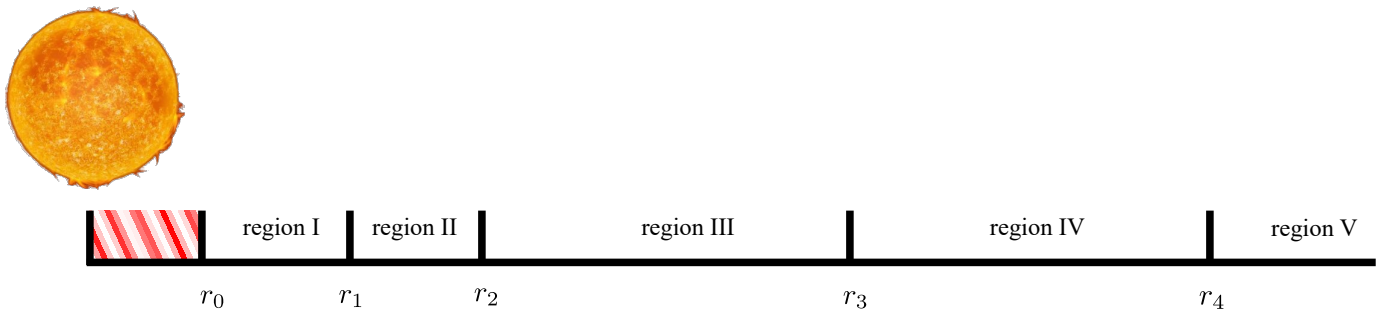
FIG. 2. There are five important regions for supercruise (SC) around a star, which depend on the distance $r$. The reddish region depicted is the exclustion zone, in which a ship drops out of SC.

were trying to fly as perpendicular as possible to the star. The reason was that $r$ would not change too quickly, and the ship had enough time to adjust its speed to be equal to $v_{\max}$. We then logged both $r$ and the velocity, which would be $v_{\max}/10$, due to our reduced throttle (see figure 1).

## III.   THE FIVE REGIONS AROUND A STAR

There are five principal regions around a star, in which its velocity curve takes on different shapes.

$$v_{\max}(r) = \begin{cases} v_{\mathrm{I}}(r) & r_0 < r < r_1 \\ v_{\mathrm{II}}(r) & r_1 < r < r_2 \\ v_{\mathrm{III}}(r) & r_2 < r < r_3 \\ v_{\mathrm{IV}}(r) & r_3 < r < r_4 \\ v_{\mathrm{V}}(r) & r_4 < r \end{cases}$$

These are, surprisingly, relatively independent of the features of the star, or the ship in question, as already indicated. Some regions depend slightly on the radius of the star, while other regions do not. All regions appear to be completely independent on the details and outfitting of the ship. In the following, we consider the five regions in detail.

### A.   Region I

Region I is the closest one in which SC is still possible. It lasts from the exclusion zone $r_0$ to the beginning of $r_1$. Throughout all of region I, the maximal velocity of a spaceship is constant, namely 33% of the speed of light.

$$v_{\mathrm{I}}(r) = \frac{1}{3} \tag{1}$$

In the cockpit, this is depicted as

$$v = 0.33c.$$

The precise position $r_0$ of the exclusion zone, as well as of $r_1$, depend on the size of the star in question. By extensive testing with various different stars, we have found that $r_1$ depends affine-linearly on the radius[5] $R$ of the star. The connection is

$$r_1 = 2.81\,R + 0.65, \tag{2}$$

where $R$ is measured in multiples of solar radii $R_{\mathrm{sol}} = 6.96 \cdot 10^8\mathrm{m} = 2.32\,ls$, and $r_1$ is given in light seconds $ls$. Figure 4 depicts the measurements.

### B.   Region II

Region II connects region I and III, and is a linear interpolation between the two. Measurements with several stars have determined the velocity curve to have the same slope of $\frac{2}{3}$ throughout, so the velocity curve is

$$v_{\mathrm{II}}(r) = \frac{2}{3}(r - r_1) + \frac{1}{3}. \tag{3}$$

The value of $r_2$ is determined by the intersection of (3) with $v_{\mathrm{III}}$, i.e. it is the solution of for

$$v_{\mathrm{II}}(r_2) = v_{\mathrm{III}}(r_2),$$

which, with (2), turns out to be

$$r_2 = \frac{10r_1 - 5}{7} = 4.01\,R + 0.21. \tag{4}$$

### C.   Region III

Region III lies between $r = r_2$ (4) and $r_3 = 76.5\,ls$. Note that, as soon as $R > 19.0 R_{\mathrm{sol}}$, then Region III

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[5] This seems counter-intuitive, since the frame-dragging effect should be influenced by mass, not by radius, and these two properties are not linearly related in a star. This just serves to show that there are many mysteries of the FSD we do not understand.
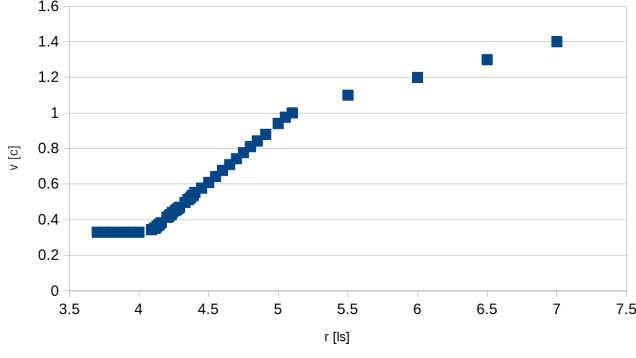
FIG. 3. Transition of the velocity curve from region I to region II to region III. The star in question has a radius of $R = 1.22 R_{\mathrm{sol}}$. This radius indicates $r_1 = 4.07$ due to (2) and $r_2 = 5.10$ due to (4), which fits the measurement data depicted in this figure.

essentially does not exist, and region II directly goes over to region IV. In region III the maximal velocity of a ship is proportional to the distance, with a good fit being

$$v_{\mathrm{III}}(r) = \frac{1}{5} r,$$

where $v_{\mathrm{III}}$ is measured in multiples of $c$, and $r$, as always, in light seconds $ls$.

### D. Region IV

Region IV is the one in which most SC will take place. It begins at

$$r_3 = 76.5, \tag{5}$$

independent of the radius of the star. The maximal velocity $v_{\mathrm{max}}$ is continuous there, i.e. $v_{\mathrm{III}}(r_3) = v_{\mathrm{IV}}(r_3)$. However, the first derivative of $v_{\mathrm{max}}$ is not continuous, as one can see in figure 5. By plotting $-\log(dv_{\mathrm{IV}}/dr)$ over $v_{\mathrm{IV}}$, one finds an affine linear relation (see figure 6), which leads to the ansatz

$$v_{\mathrm{IV}}(r) = a \ln\left(\frac{r}{b} + c\right). \tag{6}$$

By numerical regression, one can find a fairly good fit for these parameters[6], as

$$a = 351.5$$

$$b = 16826.41$$

$$c = 1.0399$$

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[6] The error between our formula and the measured values are below one percent, which we deem good enough for our calculations.
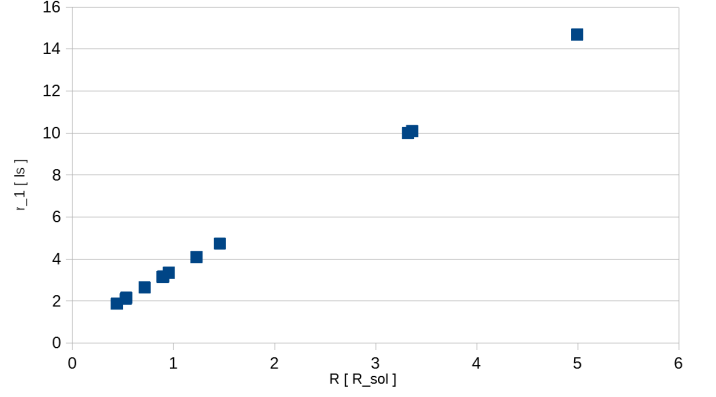


FIG. 4. Measured $r_1$ in $ls$ versus the radius $R$ of the star in solar radii $R_{\mathrm{sol}}$.
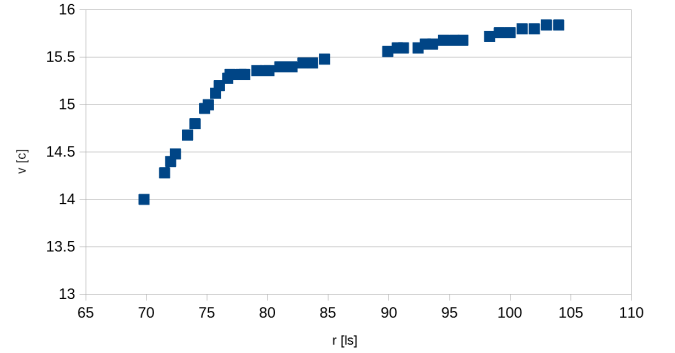


FIG. 5. Maximal velocity $v_{\mathrm{max}}$ versus $r$, around $r = r_3 = 76.5 ls$.

### E. Region V

From time to time, we all would like to travel faster than the total SC maximum speed of

$$v_{\mathrm{V}}(r) = 2001,$$

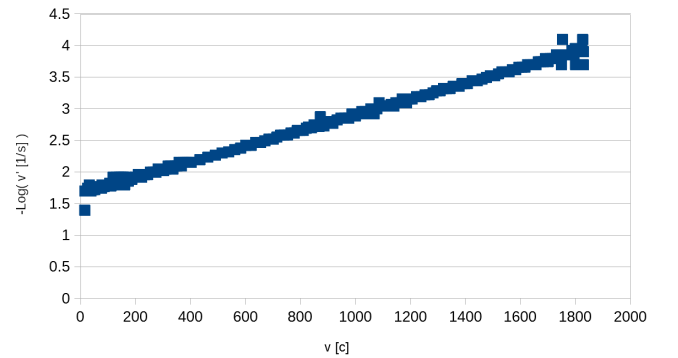but I'm afraid we can't do that. From a certain point on,



FIG. 6. Plot of $-\log(dv/dr)$ versus $v$ in region IV.

the beginning of region V, the speed maxes out. Unfortunately, a precise localisation of the beginning of region V is difficult, since the unit of $r$ in the standard HUD display of every space-ship switches from $ls$ to $ly$ as soon as $r \geq 0.1\, ly \approx 3.16 \cdot 10^6\, ls$. We therefore simply assume that $v_{\max}$ is continuous there, and define $r_4$ to be the solution to

$$v_{IV}(r_4) \;=\; 2001,$$

which leads to

$$r_4 \;=\; 4.975 \cdot 10^6\, ls.$$

This concludes the description of the maximum velocity of a ship in SC. With this information, we now turn to compute the optimal flight path.

## IV. COMPUTING THE OPTIMAL FLIGHT PATH

In order to compute the optimal flight path through a star system, we make two simplifying assumptions:

1. A ship always flies at its maximum allowed velocity $v_{\max}$.

2. A ship can change direction instantly.

Both points will be almost true for small and agile ships, but not quite as good for e.g. Cutters or Type 9s.

### A. The time-metric

For the actual computation, we use an ancient method of variational analysis and geometry. Assume a ship moves from a point with vector $\vec{r}$ to the point with vector $\vec{r} + d\vec{r}$, i.e. undergoes an infinitesimal displacement of $d\vec{r}$. The time $dt$ that passes between the two points satisfies

$$dt^2 \;=\; \frac{1}{v^2}d\vec{r}^2 \;=\; \frac{1}{v^2}\Big(dr^2 + r^2\big(d\theta^2 + \sin^2\theta\, d\phi^2\big)\Big), \quad (7)$$

where $v = v(r)$ is the velocity of the ship. We assume that

$$v(r) \;=\; v_{\max}(r),$$

which only depends on $r$. We work with polar coordinates $x^i = (r, \phi, \theta)$ in $\mathbb{R}^3$, so we assume that the only star sits in the center of the coordinate system.

We can see that the notation (7) is similar to that of a metric of Riemannian signature, with time $t$ playing the role of the arc length of a curve. We therefore call it "time-metric", or "timetric" for short. Minimising the arc length of curves amounts to choosing the routes which

have stationary time – which, for Riemannian metrics, are also those of local minimal time[7].

To compute the curves of minimal arc lenght (i.e. minimal time needed to travel along the curves), one can solve the geodesic equations for the metric (7). These equations are [3]:

$$\ddot{x}^i \;+\; \Gamma^i{}_{jk}\dot{x}^j\dot{x}^k \;=\; 0, \quad (8)$$

where the dot $\cdot$ refers to the derivative by the curve parameter $t$. Equations (8) are the geodesic equations for the time-metric (7), with the Christoffel symbols

$$\Gamma^i{}_{jk} \;=\; \frac{1}{2}g^{il}\left(\frac{\partial g_{lj}}{\partial x^k} + \frac{\partial g_{lk}}{\partial x^j} - \frac{\partial g_{jk}}{\partial x^l}\right). \quad (9)$$

Here $g_{ij}$ denote the coefficients of the time-metric, which one can read off from (7) as

$$g_{rr} \;=\; \frac{1}{v^2}, \quad g_{\theta\theta} \;=\; \frac{r^2}{v^2}, \quad g_{\phi\phi} \;=\; \frac{r^2\sin^2\theta}{v^2}.$$

The $g^{ij}$ denote the coefficients of the inverse time-metric, which are given by

$$g^{rr} = \frac{1}{g_{rr}}, \quad g^{\theta\theta} \;=\; \frac{1}{g_{\theta\theta}}, \quad g^{\phi\phi} = \frac{1}{g_{\phi\phi}}.$$

It is straightforward to compute the Christoffel symbols:

$$\Gamma^r{}_{rr} \;=\; \frac{v'}{v}, \qquad\qquad \Gamma^r{}_{\theta\theta} \;=\; r^2\frac{v'}{v} - r$$

$$\Gamma^r{}_{\phi\phi} \;=\; \sin^2\theta\,\Gamma^r{}_{\theta\theta}, \qquad \Gamma^\theta{}_{\theta r} \;=\; \Gamma^\theta{}_{r\theta} = \frac{1}{r} - \frac{v'}{v}$$

$$\Gamma^\phi{}_{\phi r} \;=\; \Gamma^\phi{}_{r\phi} \;=\; \frac{1}{r} - \frac{v'}{v}$$

$$\Gamma^\theta{}_{\phi\phi} \;=\; -\sin\theta\,\cos\theta \qquad \Gamma^\phi{}_{\phi\theta} \;=\; \Gamma^\phi{}_{\theta\phi} = \tan\theta$$

By $v'$ we denote the derivative of $v$ w.r.t. $r$.

The metric (7) has three Killing vector fields $L_1$, $L_2$, $L_3$, which satisfy the standard rotation algebra $[L_i, L_j] = \epsilon_{ijk}L_k$ (+cyclic permutations). Therefore, there is a conserved 3-vector along geodesics. Without loss of generality, one can set $\theta \equiv \frac{\pi}{2}$, i.e. only treat motion in the $(x, y)$-plane. With that choice, the conserved vector points in the $z$-direction, and its norm is equal to the inner product $\ell := \langle \dot{x}, L_3 \rangle = \langle \dot{x}, \partial_\phi \rangle$, given by

$$\ell \;=\; \frac{r^2}{v(r)^2}\dot{\phi}. \quad (10)$$

The quantity (10) is the analogue of the angular momentum, although weighted by $v(r)^2$.

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[7] "Local" here means that the curves determined by this method cannot be changed without increasing total flight time $t$. However, there might be other local minima with smaller $t$.

Since $\theta$ is fixed, the only interesting dynamical quantities ar $r$ and $\phi$, and their equations of motion are

$$\ddot{r} + \frac{v'}{v}\dot{r}^2 + \left(r^2\frac{v'}{v} - r\right)\dot{\phi}^2 = 0, \qquad (11)$$

$$\ddot{\phi} + 2\left(\frac{1}{r} - \frac{v'}{v}\right)\dot{\phi}\dot{r} = 0. \qquad (12)$$

The second geodesic equation (12) can be rewritten as

$$\frac{d\ell}{dt} = 0, \qquad (13)$$

which is just the conservation of $\ell$, as expected. The equation (11) can, together with (12), be reformulated as

$$\frac{d}{dt}\left(\frac{\dot{r}^2 + r^2\dot{\phi}^2}{v^2}\right) = 0, \qquad (14)$$

which is just the normalisation of the curve, given by

$$\dot{r}^2 + r^2\dot{\phi}^2 = v^2, \qquad (15)$$

which one can, with (10), rewrite as

$$\dot{r} = \pm v\sqrt{1 - \frac{\ell^2 v^2}{r^2}}. \qquad (16)$$

Here one has to be careful as to choose the correct sign, which is determined by the initial conditions of the curve (and decides whether one approaches the star, or moves away from it). In order to solve the geodesic equations for a given initial position and initial velocity, one way is to compute $\ell$ from the initial data as per (10), and then solve the ODE (16), which gives $r(t)$. Then one can rewrite (10) as

$$\dot{\phi} = \frac{\ell v^2}{r^2} \qquad (17)$$

with the solution for $r(t)$, and integrate it to find $\phi(t)$. In the following, we will go a slightly different path, since we will be interested not in the initial value problem, but rather the boundary data problem.
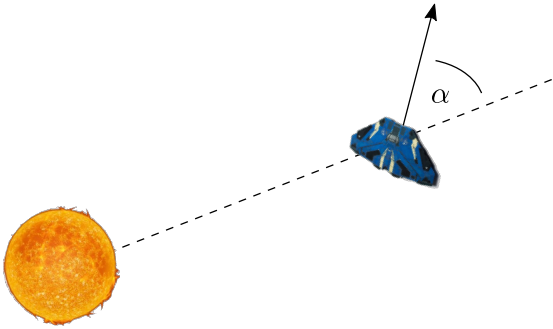


FIG. 7. Definition of the flight angle $\alpha$.

An important concept for studying the dynamics of flight paths will be the flight angle $\alpha$, measured as angle between the direction of flight and the outward normal pointing away from the star (see figure 7). From elementary geometry, we have

$$\sin\alpha = \frac{r\dot{\phi}}{v}. \qquad (18)$$

### B. Flight paths in region I

Since $v$ is constant in region I, the solution to the geodesic equations (8) are simply straight lines. So apart from the exclusion zone at $r = r_0$, one should try to reach ones destination as quickly as possible when flying on a straight line.

### C. Flight paths in region II

Consider the equation (11) for $r$. In region II, one has

$$\frac{v'}{v} - \frac{1}{r} = \frac{-1.15 + 2.81R}{r(1.15 + r - 2.81R)} \qquad (19)$$

For $R > 0.409$, this is always greater than zero, which means that $\ddot{r} < 0$ for any flight angle. This means that all flight paths are curved back to the star. In order to leave region II, a geodesic therefore needs to have a low enough flight angle $\alpha$.

This is depicted in figure 8. Left we have a flight path with low $\alpha_1$, which can leave region II and enter region III, while the high flight angle $\alpha_1$ on the right leads to a geodesic which curves back to the star, never reaching region III. The boundary between the two cases is when $\alpha_1 = \alpha_{\text{crit}}$, i.e. when $\alpha_2 = \frac{\pi}{2}$. Define

$$v_1 := v_{\text{II}}(r_1) = \frac{1}{3}$$

$$v_2 := v_{\text{II}}(r_2) = \frac{1}{5}r_2.$$

Due to (10) and (18), we have

$$\ell = \frac{r_1}{v_1}\sin\alpha_1 = \frac{r_2}{v_2}\sin\alpha_2,$$

in other words

$$\sin\alpha_{\text{crit}} = \frac{5}{3r_1}. \qquad (20)$$

The critical angle depends on $r_1$, and therefore, via (2), on the radius $R$ of the star. The relation between the two is depicted in figure 9.

As one can see, the critical angle decreases quickly. Geodesics which have a flight angle lower than the critical value $\alpha_{\text{crit}}$ cannot leave region II. In other words, traversing region II on an optimal flight path requires a flight angle smaller than the critical one. In general one can say:
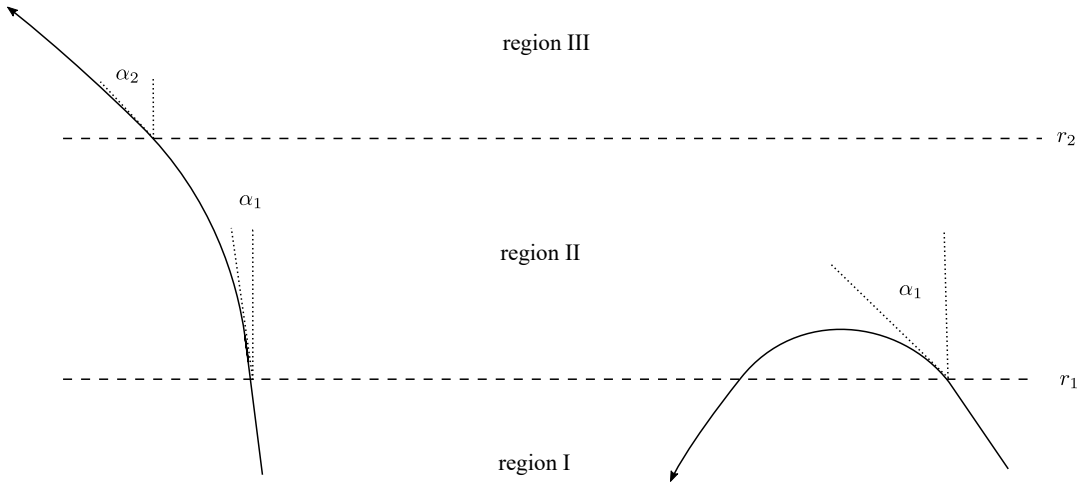
FIG. 8. When $\alpha_1$ at $r = r_1$ is larger than the critical angle $\alpha_{\text{crit}}$, the geodesic can not reach $r_2$, and is deflected back to region I. This is depicted in the right case. In the left case, $\alpha_1$ is lower than $\alpha_{\text{crit}}$, which allows the geodesic to reach region III.



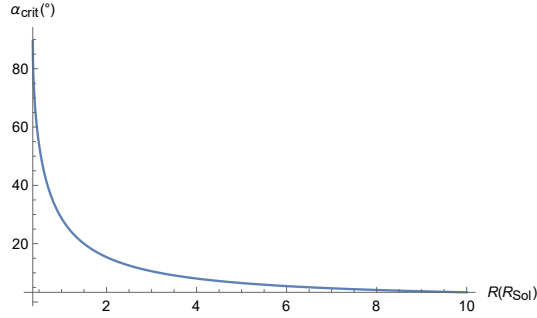FIG. 9. Critical angle $\alpha_{\text{crit}}$ depicted for different star radii $R$.

> *When you need to get further away from the star than a few light seconds, do so as directly as possible!*

In other words, keep the flight angle $\alpha$ smaller than the critical value $\alpha_{\text{crit}}$. A low flight angle means flying almost directly away from the star. Of course, the exact flight angle depends very much on where one wants to go.

### D. Flight paths in region III

In region III, the optimal flight path is of a quite simple form, since $v(r) = A \cdot r$ with

$$A = \frac{1}{5}.$$

Therefore, the motion for $\phi$ can be read off the conservation law (10), which then reads

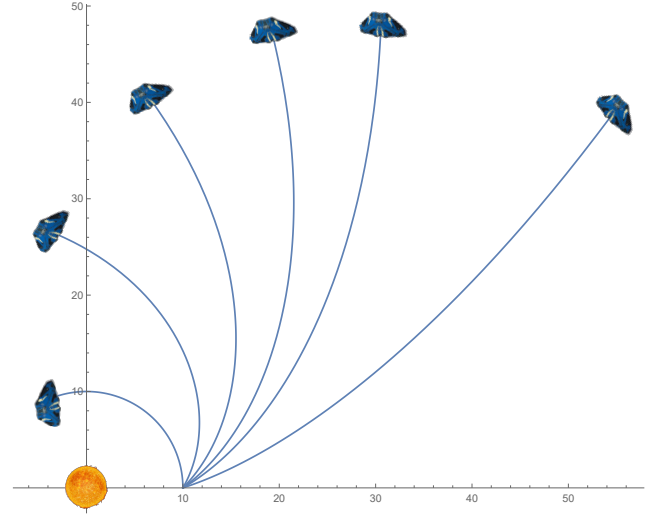$$\dot{\phi} = A^2 \ell = \text{const},$$



FIG. 10. Flight paths in region III around a star: Several logarithmic spirals, which start at $r = 10\,ls$, and with different values for $\alpha = \pi/10, \pi/6, \pi/5, \pi/4, \pi/3, \pi/2$. Axes labeled by $ls$. Star and ships not to scale.

i.e. $\dot{\phi}$ is constant. On the other hand, consider the angle $\alpha$ between the velocity vector and the outward pointing normal (see figure 7), we then get

$$\sin \alpha = \frac{r}{\phi} v = \frac{1}{A} \dot{\phi}.$$

In other words, the geodesics are those curves which have a constant angle with the outward normals. These are called *logarithmic spirals*. The solution can be readily computed: A flight path starting at the coordinates $(r_0, \phi_0)$ with a flight angle $\alpha$ leads to

$$r(t) = r_0 \exp\left(A\,t\,\cos\alpha\right),$$

$$\phi(t) = \phi_0 + A\,t\,\sin\alpha.$$

Different flight paths for varying $\alpha$ are depicted in figure 10. The rule for flight paths entirely in region III can thus be formulated as follows:

> *When flying in region III, keep a constant angle towards the star.*

While sounding simple enough, in practice it might prove difficult, since the necessary value of the flight angle $\alpha$ that one should keep depends on where one wants to go. Still, as a rule of thumb, one could say that, when trying to reach a point on the other side of the star, which has a roughly similar distance to the star than the own ship, it is most time-efficient to keep a constant distance to the star, and travel to the desired point on a circular arc. This could be formulated as follows:

> *In region III, the quickest path to the other side of a star is the circular path around it.*

### E. Flight paths in region IV

Region III, which ranges from $r = r_3 = 76.5ls$ to $r = r_4$, is the region in which most flight will realistically take place. The form (6) of $v(r)$, however, makes it difficult to solve the geodesic equations analytically. To get some feeling for the geodesics that run in this region, We consider two cases:

Case 1: A ship starts at a distance of $r = r_0$ to the star. What is the quickest path to the opposite side of the star with the same distance?

To compute this numerically, we rewrite the equation for $\dot{r}$ as a differential equation for $r(\phi)$. We get, with (16):

$$\left(\frac{dr}{d\phi}\right)^2 = \frac{\dot{r}^2}{\dot{\phi}^2} = \frac{v^2 - r^2\dot{\phi}^2}{\dot{\phi}^2}.$$

We assume that the ship begins its journey at $\phi = 0$ and travels to $\phi = \pi$, where $r = r_0$ in both cases. Due to symmetry, the path needs to have $\dot{r} = 0$ at $\phi = \frac{\pi}{2}$. At that point, the ship is closest to the star, i.e. $r = r_{\min}$. With (14) one can deduce

$$\ell = \frac{r_{\min}^2 \dot{\phi}}{v_{\min}^2} = \frac{r_{\min}}{v_{\min}}$$

with $v_{\min} = v(r_{\min})$. Hence, we have

$$\left(\frac{dr}{d\phi}\right)^2 = r^2\left(\frac{v_{\min}^2 r^2}{v^2 r_{\min}^2} - 1\right).$$

| $r_{\min}$ | $r_0$ | $r_{\min}$ | $r_0$ |
|---|---|---|---|
| 100 | 528 | 8000 | 10971 |
| 200 | 697 | 8500 | 11715 |
| 300 | 846 | 9000 | 12471 |
| 400 | 984 | 9500 | 13236 |
| 500 | 1116 | 10000 | 14011 |
| 750 | 1430 | 20000 | 31327 |
| 1000 | 1733 | 30000 | 51375 |
| 2000 | 2930 | 40000 | 73460 |
| 3000 | 4123 | 50000 | 97164 |
| 4000 | 5421 | 75000 | 161883 |
| 5000 | 6738 | 100000 | 232216 |
| 6000 | 8103 | 180000 | 485051 |
| 7000 | 9515 | 300000 | 911453 |

FIG. 11. Pairs of $r_{\min}$ and $r_0$ in region IV. All numbers are measured in $ls$.

For $\phi(r)$ this gives an ordinary differential equation

$$\frac{d\phi}{dr} = \pm \frac{1}{r\sqrt{\frac{v_{\min}^2 r^2}{v^2 r_{\min}^2} - 1}},$$

which one can solve numerically, e.g. with $\phi(r_{\min}) = \frac{\pi}{2}$ as initial condition. This way, one can restrict to the $+$-sign for the whole trajectory. The point $r_0$ can then be read off as the solution of $\phi(r_0) = \pi$. The table in figure 11 depicts several pairs $r_0$, $r_{\min}$, and figure 12 shows the nontrivial dependence of the two. Some sample flight paths have been drawn in figures 15 and 16.

One can see that $r_0/r_{\min}$ has a minimum at around $r_0 \approx 10^{3.75}ls$. This is interesting, in that it means the decision of how to get to the opposite side of a star depends on how far one is away initially. If one is at around $r_0 \approx 7500ls - 1000ls$, it is quickest to travel roughly a circular path around the star. However, if one is much further away or much closer, it is more efficient to pass the star much closer, i.e. let $r_{\min}$ be much smaller than $r_0$.

Case 2: We consider a ship flying from $A$ to $B$, both at distance $1000ls$ to the star, and separated by an angle of $\Delta\phi$.

From the previous discussion, we know that if $\Delta\phi = \pi$, then $r_{\min} \approx 412ls$. The different $r_{\min}$ can be read off in table 14. Some trajectories are depicted in figure 13.

In general one can say that it is beneficial to not get too close to the star. Of course, in such generality the statement is not very helpful. However, the precise details of how close one should get to the star at all depends very much on the boundary conditions, i.e. where one is and where one want to go. Thus, there is no universal statement for how to travel in region III. The best one can give is some general advice.
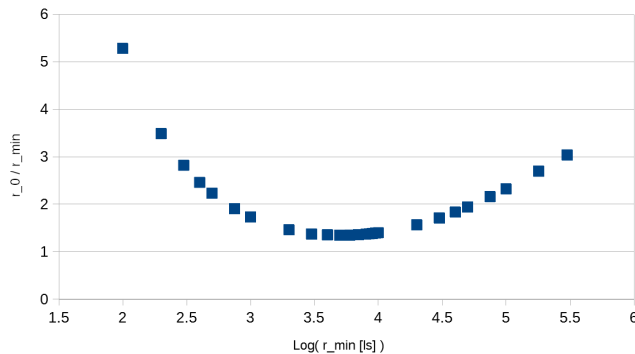
FIG. 12. The ration $r_0/r_{\min}$ has a minimum at around $r_0 \approx 10^{3.75}$, at which it becomes around $r_0/r_{\min} \approx 1.3$. These curves are almost circular, while curves with different $r_0$ are much flatter. See also figures 15, 16 for examples.

> *On your path through region IV, do not get much closer to the star if possible. Your minimal distance to the star should not be lower than one half to one quarter of your initial position.*

### F.   Flight paths in region V

The speed is constantly $v = 2001$ in region V, so optimal paths are straight lines again. Since very few points of interest usually lie in region V, a further discussion is unnecessary.

## V.   SUMMARY

In this article we have presented measurements of the maximal velocity $v_{\max}$, i.e. the speed a ship attempts to reach when at full throttle. We found that this speed does only depend on the distance to the nearest star, but does *not* depend on the type of ship or its modules, and almost not at all on the details of the star in question.

We found that the dependence of $v_{\max}$ on $r$ is behaving vastly differently in different distances to the star. We have denoted these regions from I to V (figure 2).

With the measurement of $v_{\max}$, we could compute optimal flight paths through a system with only one central star and no planets. We did this by computing solutions to the geodesic equations (11), (12). Naturally, the dynamics is very different depending on which region the ship is in, i.e. how far away it is precisely from the star.

The details of optimal flight paths can be found in the article. In the text, we formulated some guidelines for describing the geodesics. They are specific to the regions, and can be summarised as the following "rules for optimal flying":

- *Rule 1: If you are close to the star, get away, and leave the region where $v = 0.33c$ as directly as possible!*

- *Rule 2: It is almost never recommendable to fly directly towards one's target in a straight line. Rather, give the nearest star a wide berth*

These rules are, of course, only guidelines. Also, they probably amount to what many CMDRs already do intuitively. Nonetheless, we hope that this quantitative analysis might be of benefit for some pilots out there.

Fly Safe!

[1] M. Alcubierre, "The Warp drive: Hyperfast travel within general relativity," *Class. Quant. Grav.*, vol. 11, pp. L73–L77, 1994.

[2] M. Alcubierre and F. S. N. Lobo, "Warp Drive Basics," *Fundam. Theor. Phys.*, vol. 189, pp. 257–279, 2017.

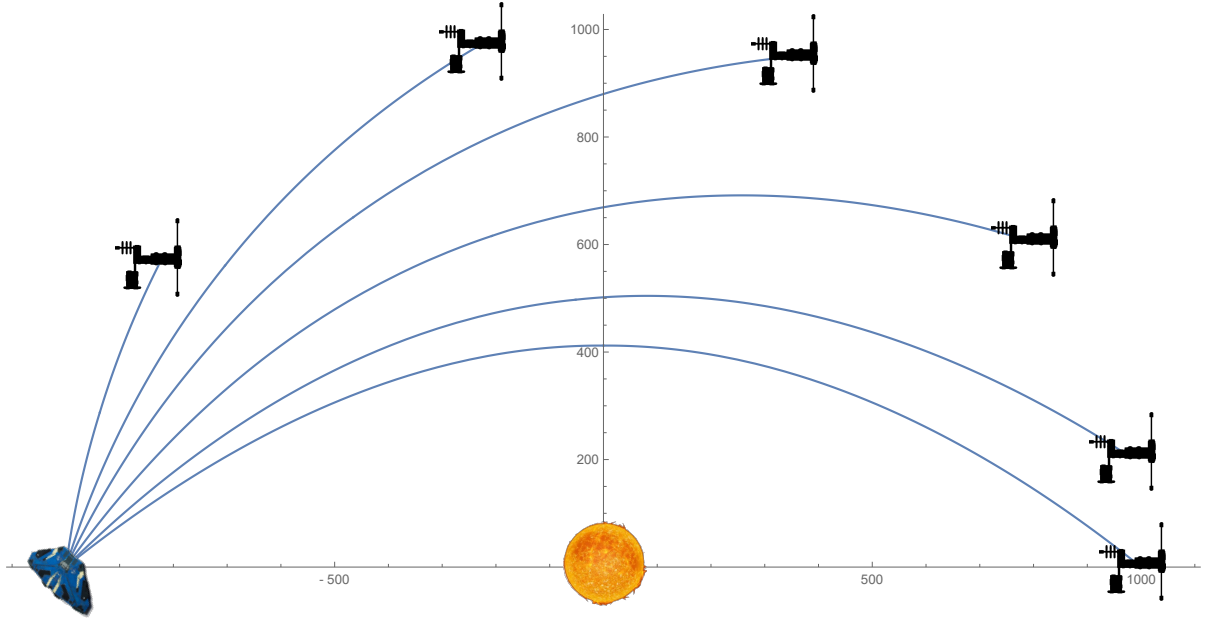[3] B. Schutz, *Geometrical Methods of Mathematical Physics.* Cambridge University Press, 1980.

FIG. 13. Different flight paths in region IV. The ship starts from the point on the left, $1000\,ls$ away from the star, and travels to different stations, having the same distance to the star, but with some separation angle $\Delta\phi$ (i.e. the angle between ship and station, as seen from the star). The flight paths are accurate solutions to the geodesic equations (11), (12). Axes labeled by $ls$. Star, ship, and stations not to scale.

| $\Delta\phi$ | $r_{\min}$ | $\Delta\phi$ | $r_{\min}$ |
|---|---|---|---|
| 178.2 | 425 | 54.4 | 950 |
| 175.3 | 450 | 48.7 | 960 |
| 167.9 | 500 | 41.8 | 970 |
| 151.8 | 600 | 34.4 | 980 |
| 132.4 | 700 | 24.0 | 990 |
| 108.9 | 800 | 16.6 | 995 |
| 94.5 | 850 | 0 | 1000 |
| 77.3 | 900 | | |

FIG. 14. Pairs of separation angle $\Delta\phi$ (measured in $°$), and $r_{\min}$ (in $ls$) for paths in region III.
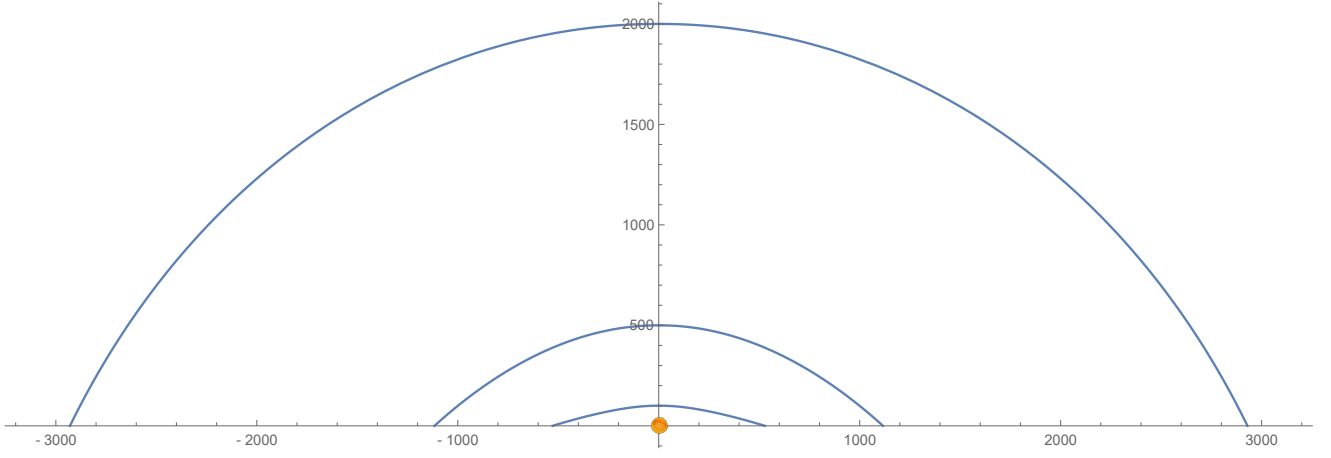
FIG. 15. Three optimal flight paths from one side of the star to the other, with different minimal distances $r_{\min} = 100ls$, $500ls$, and $2000ls$. The star is situated in the center of the coordinate system, but is not to scale. Axes labeled by $ls$.
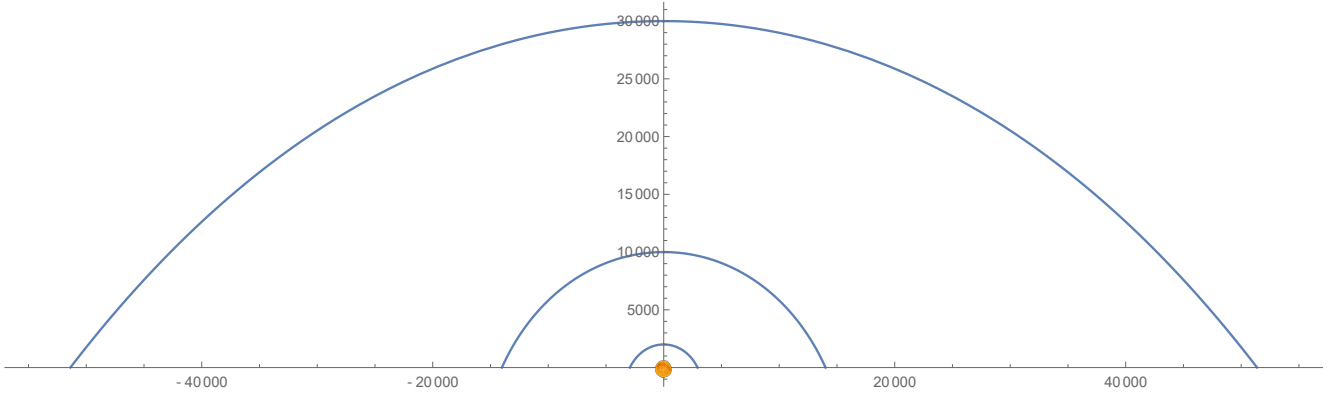


FIG. 16. Three optimal flight paths from one side of the star to the other, with different minimal distances $r_{\min} = 2000ls$, $1000ls$, and $30000ls$. The star is situated in the center of the coordinate system, but is not to scale. Axes labeled by $ls$.